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THE BEHAVIOR OF AN ELECTRON PLASMA WITH AN ARBITRARY DEGREE OF DEGENERACY OF THE ELECTRON GAS IN HALF-SPACE WITH DIFFUSE BOUNDARY CONDITIONS

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Abstract. We analytically solve a boundary value problem for the behavior (oscillation) of an electron plasma with an arbitrary degree of degeneracy of the electron gas in half-space with mirror boundary conditions. We apply the Vlasov–Boltzmann kinetic equation with a collision integral of the Bhatnagar–Gross–Krook type and a Poisson equation for the electric field. We analyze the behavior of the electric field near interfaces for frequencies close to the plasma oscillation frequency.

Keywords: *Vlasov–Boltzmann equation, collision frequency, electric field, Drude mode, Debye mode, van Kampen mode, dispersion function*

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1. Introduction

The character of electric field screening near the surface of a conductor is critically important for different problems of surface physics (Girard *et al*., 2000; Keller, 1996; Suleimanova & Yushkanov, 2020; Yushkanov & Zverev, 2017, Lifshitz & Pitaevsky, 1979), in particular, the problem of propagation of plasma oscillations (Bozhevolnyi, 2008; Pitarke *et al*., 2017).

The purpose of the manuscript is to study the system of electron–plasma response with an arbitrary degree of degeneracy to an external alternating electric field.

Here, we have obtained an analytical solution to the problem on the behavior of a semi-infinite plasma with an arbitrary degree of electron gas degeneracy in an external ac electric field perpendicular to the plasma surface. Such a situation takes place, e.g., when analyzing a solid-state semiconductor plasma. We use the Vlasov–Boltzmann kinetic equation with the Bhatnagar–Gross–Krook (BGK) collision integral for the electron distribution function and Poisson equation for the electric field.

It makes it possible to separate energy absorption into the volume and surface components. Surface absorption is analyzed in detail. A nontrivial character of the dependence of surface absorption on the ratio between the volumetric electron collision frequency and the frequency of the external electric field is demonstrated.

2. Formulation of the problem and basic equations

The general statement of the problem is given in (Latyshev & Suleimanova, 2018). We will use the τ-model Vlasov–Boltzmann equation:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} = \nu\left(f_{eq} - f\right) \tag{1}$$

The behavior of the electric field in plasma is described by Poisson equation

$$\mathrm{div}\mathbf{E} = 4\pi\rho, \quad \rho = e\int (f - f_0)d\Omega_F\,, \;\; d\Omega_F = \frac{(2s+1)d^3p}{(2\pi\hbar)^3} \tag{2}$$

Here, f is the electron distribution function; f_{eq} is the locally equilibrium Fermi–Dirac distribution function, $f_{eq}(\mathbf{r}, v, t) = \left\{1 + exp\frac{\mathcal{E} - \mu(\mathbf{r},t)}{kT}\right\}^{-1}$, $f_0 = f_{FD}$ is the unperturbed Fermi–Dirac distribution function, $f_0(v, \mu_0) = f_{FD}(v, \mu_0) = \left\{1 + exp\frac{\mathcal{E} - \mu_0}{kT}\right\}^{-1}$, $\mathbf{p} = m\mathbf{v}$ is the electron momentum; $\mathcal{E} = mv^2/2$ is the electron kinetic energy; $\mu_0 = const$ and $\mu(\mathbf{r}, t)$ are the unperturbed and perturbed chemical potentials, respectively; e and m are the charge and effective mass of an electron, respectively; ρ is the charge density; $\hbar$ is Planck's constant; ν is the electron scattering frequency; s is the particle spin ($s = 1/2$ for electrons); k is the Boltzmann constant; T is the plasma temperature, which is assumed to be constant; and $\mathbf{E}(\mathbf{r}, t)$ is the electric field in plasma.

Let us consider the condition of diffusive reflection of electrons from the boundary of a semi-infinite plasma: $f(x = 0, \mathbf{v}, t) = f_{\mathrm{eq}}(x = 0, \mathbf{v}, t)$ at $v_x > 0$, $e(0) = 1$, $e(+\infty) < +\infty$. The external electric field on the plasma surface is perpendicular to the plasma boundary and varies in time as $\mathbf{E}_{ext}(t) = E_0 e^{-i\omega t}(1,0,0)$.

The corresponding self-consistent electric field in plasma has the form $\mathbf{E}(x, t) = E(x)e^{-i\omega t}(1,0,0)$.

We assume that the external field is sufficiently weak, so that the linear approximation is applicable. Eqs. (1) and (2) can be linearized with respect to the absolute Fermi–Dirac distribution function f_0: $f_{eq}(x, P, t) = f_0(P, \alpha) + g(P, \alpha)\delta\alpha(x)e^{-i\omega t}$, where $f_0(P, \alpha) = f_{FD}(P, \alpha) = \left(1 + e^{P^2 - \alpha}\right)^{-1}$, $g(P, \alpha) = e^{P^2 - \alpha}/\left(1 + e^{P^2 - \alpha}\right)^2$, $\mathbf{P} = \mathbf{p}/p_T = \mathbf{v}/v_T$. Here v_T is the electron thermal velocity given by $v_T = \sqrt{2kT/m}$ and $\alpha = \mu/kT$ is the reduced chemical potential. The change of the chemical potential is considered to be a small parameter so that representation $\alpha(x, t) = \alpha + \delta\alpha(x)e^{-i\omega t}$ is possible. We linearize the electron distribution function $f(x, P, P_x, t) = f_0(P, \alpha) + g(P, \alpha)h(x, P_x)e^{-i\omega t}$, where $h(x, P_x)$ is a new unknown function and $h(x, P_x) \sim E$.

As a result, we get a system containing new unknown functions and dimensionless variables. The detailed solution is given in (Suleimanova & Yushkanov, 2018). The solution is based on the method of separation of variables, is reduced to obtaining the dispersion function and search eigenfunctions by which we can decompose the resulting analytical solution. Dispersion function determines the range of solutions to the problem

$$\Lambda(z) = 1 - \frac{1}{w_0} - \frac{z^2 - \eta_1^2}{w_0\eta_1^2}\lambda_0(z, \alpha),$$

$$\lambda_0(z,\alpha) = 1 + z \int\limits_{-\infty}^{+\infty} \frac{k(\mu,\alpha)d\mu}{\mu - z}.$$

Constants w_0, η_1^2 and function $k(\eta,\alpha)$ have forms

$$f_0(\eta,\alpha) = \frac{1}{1+e^{\eta^2-\alpha}}, \qquad k(\eta,\alpha) = \frac{f_0(\eta,\alpha)}{2s_0(\alpha)},$$

$$s_0(\alpha) = \int\limits_0^{+\infty} f_0(t,\alpha)dt, \qquad s_2(\alpha) = \int\limits_0^{+\infty} t^2 f_0(t,\alpha)dt,$$

$$w_0 = 1 - i\frac{\omega}{\nu}, \qquad \eta_1^2 = w_0 \frac{\nu^2}{\omega_p^2}\frac{s_2(\alpha)}{s_0(\alpha)},$$

$\Omega = \omega/\omega_p$, $\varepsilon = \nu/\omega_p$, ω_p is the plasma (Langmuir) frequency, $\omega_p = \sqrt{4\pi e^2 N/m}$, and N is the equilibrium electron number density (concentration).

As a result of the solution, the induced electromagnetic field is represented as the sum of three terms corresponding to the expansion in the spectrum of the dispersion function. In general, the structure of an electric field arising in a plasma can be represented as $e(x) = e_d(x) + e_c(x)$,
where

$$e_d(x) = E_\infty + E_d \exp\left(-\frac{w_0 x}{\eta_0}\right), \tag{3}$$

$$e_c(x) = \int\limits_0^{\infty} \frac{1}{2\pi i(\eta^2 - \eta_1^2)}\left(C_0 + \frac{C_{-1}}{\eta - \eta_0}\right)\left(\frac{1}{X^+(\eta)} - \frac{1}{X^-(\eta)}\right)\exp\left(-\frac{w_0 x}{\eta_0}\right)d\eta. \tag{4}$$

Here

$$E_\infty = C_0 = \frac{\Lambda_1}{\Lambda_\infty}, \qquad E_d = \frac{C_0(\eta_1/(\eta_0^2 - \eta_1^2) + \alpha^-)}{X(\eta_0)(\eta_1\alpha^+ - \eta_0\alpha^-)},$$

$$\Lambda_1 = \Lambda(\eta_1) = 1 - \frac{1}{w_0}, \qquad \Lambda_\infty = \Lambda(\infty) = 1 - \frac{1}{w_0} + \frac{1}{w_0^2\varepsilon^2},$$

$$C_{-1} = -\frac{C_0[\eta_1 + \alpha^-(\eta_0^2 - \eta_1^2)]}{\eta_1\alpha^+ - \eta_0\alpha^-}, \qquad \alpha^\pm = \frac{X(\eta_1) \pm X(-\eta_1)}{2},$$

$$X(z) = \frac{1}{z}\exp V(z), \qquad V(z) = \frac{1}{\pi}\int\limits_0^{\infty}\frac{\zeta(\tau)d\tau}{\tau - z},$$

$$\zeta(\tau) = \frac{1}{2i}\ln G(\tau) - \pi.$$

The detailed description of the function $G(\tau)$ is given in (Latyshev & Suleimanova, 2018; Suleimanova & Yushkanov, 2018).

Here, $e_d(x)$ corresponds to the discrete spectrum and $e_c(x)$ corresponds to the continuous spectrum.

The analysis below is based on the idea of seeking the zero η_0 of the dispersion function in explicit form. As $\omega \to \omega_p$ and $\varepsilon \to 0$, it turns out that $\eta_0(\Omega,\varepsilon) \to \infty$. We use the expansion of the dispersion function for $|\eta_0| > 1$. Neglecting terms starting from the order η^{-6} in the Laurent expansion of the dispersion function, from the equation $\Lambda(\eta_0) = 0$, we obtain

$$\eta_0^2 = -\frac{(\Omega + i\varepsilon)^2}{\Omega(\Omega + i\varepsilon) - 1}\Lambda_2(\alpha) + \frac{\Lambda_4(\alpha)}{\Lambda_2(\alpha)},$$

where

$$\Lambda_2(\alpha) = \frac{1}{(\varepsilon - i\Omega)^2}\frac{s_4(\alpha)}{s_2(\alpha)} - \frac{\varepsilon}{\varepsilon - i\Omega}\frac{s_2(\alpha)}{s_0(\alpha)},$$

$$\Lambda_4(\alpha) = \frac{1}{(\varepsilon - i\Omega)^2}\frac{s_6(\alpha)}{s_2(\alpha)} - \frac{\varepsilon}{\varepsilon - i\Omega}\frac{s_4(\alpha)}{s_0(\alpha)}.$$

We consider w_0/η_0 and w_0 with $\Omega = 1$ as $\varepsilon \to 0$:

$$\frac{w_0}{\eta_0} = \frac{1-i}{\sqrt{2}} \cdot \sqrt{\frac{s_2(\alpha)}{\varepsilon s_4(\alpha)}}, \qquad w_0 = -\frac{i}{\varepsilon}. \tag{5}$$

It is clear from expression (3) for the discrete spectrum with asymptotic formula (5) taken into account that the corresponding part of the electric field has coefficient of decay in x proportional to $\left(\sqrt{\varepsilon}\right)^{-1}$. It is clear from expression (4) for the continuous spectrum with the asymptotic w_0 taken into account that the corresponding part of the field has decrement decrement proportional to ε^{-1}. This means that there are two layers $0 \leq x \leq \varepsilon$ and $\varepsilon \leq x \leq \sqrt{\varepsilon}$ adjacent to the plasma boundary. In the first layer, we should take the contribution to the electric field determined by both the continuous and the discrete spectra into account. In the second layer, the decisive contribution to the electric field comes from the second term in (3) with the Debye amplitude. Second layer with $x \sim \sqrt{\varepsilon}$ passes into the domain of a continuous medium, where the determining contribution to the electric field comes from the first term with the Drude amplitude.

Passing to dimensional coordinates, we find that the first layer corresponds to the domain $0 \leq x \leq l\varepsilon$ and the second layer corresponds to the domain $l\varepsilon \leq x \leq l\sqrt{\varepsilon}$. Taking the definition of ε into account, we obtain $0 \leq x \leq r_D$ for first layer and $r_D \leq x \leq \sqrt{lr_D}$ for second layer. Here r_D is the Debye screening radius of the field.

For large $|\eta_0|$, the magnitude of the electric field corresponding to the discrete spectrum is equal to

$$e_d(x) = \frac{\Lambda_1}{\Lambda_\infty} - \frac{\Lambda_1}{\Lambda_\infty}\exp\left(-\frac{w_0 x}{\eta_0} - V(\eta_0)\right),$$

Considering that $|\eta_0| \gg 1$

$$V(\eta_0) = \frac{V_1}{\eta_0} + \frac{V_2}{\eta_0^2} + \cdots,$$

where

$$V_n = -\frac{1}{2\pi i}\int_0^\infty [\ln G(\tau) - 2\pi i]\, \tau^{n-1} d\tau, \qquad n = 1,2, \ldots$$

This gives

$$\exp\left(-V(\eta_0)\right) = 1 - \frac{V_1}{\eta_0} + \cdots.$$

The electric field corresponding to the discrete spectrum is calculated by the formula

$$e_d(x) = \frac{\Lambda_1}{\Lambda_\infty}\left(1 - e^{-\frac{w_0 x}{\eta_0}}\right) + \frac{V_1\Lambda_1}{\eta_0\Lambda_\infty} e^{-\frac{w_0 x}{\eta_0}}.$$

Consequently, we have $e_d(x) = \frac{V_1\Lambda_1}{\eta_0\Lambda_\infty}$ at the plasma boundary with $|\eta_0| \gg 1$. Taking into account the boundary condition on the field $e_d(0) + e_c(0) = 1$, we obtain that $e_c(0) = 1 - \frac{V_1\Lambda_1}{\eta_0\Lambda_\infty}$ at $|\eta_0| \gg 1$.

Thus, in the first layer, the contributions to the electric field of the discrete and continuous spectra are comparable in magnitude. This means that the contribution of the continuous spectrum near the surface (in the first layer) should be taken into account, because both quantities $e_d(0)$ and $e_c(0)$ have the same order at $\eta_0 \to \infty$.

3. Conclusion

In this work, we analyze the behavior of the electric field near the interfaces for frequencies close to the frequency of plasma oscillations.

We analyzed the dependence of the domain where the Debye mode exists on problem parameters such as the chemical potential of the electron gas and the electron collision frequency.

We investigated the case where the frequency of the external field oscillations is close to the resonant frequency of plasma oscillations. The investigation showed that a layer of the width $0 \le x \le r_D$ is adjacent to the plasma surface. In this layer, the field behavior is determined by three terms: the Drude and Debye modes (they both correspond to the discrete spectrum of the problem) and the van Kampen mode (it corresponds to the continuous spectrum). A second layer has the width $r_D \le x \le \sqrt{lr_D}$. In this layer, the field behavior is determined by Drude and Debye modes, and the Debay mode is main characteristic responsible for the oscillation regime.

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